African Research Review

An International Multi-Disciplinary Journal

*ISSN 1994-9057 (**Print**)* *ISSN 2070-0083 (**Online**)*

Volume 2 (4) September, 2008 **Special Edition: Engineering**

Practical Recommendations for the Preliminary Design Analysis of Multi-Story Moment-Resisting Frames (pp.17-30)

C. Arum - Department of Civil Engineering, Bahir Dar University, Bahir Dar, Ethiopia. arumcnwchrist@yahoo.co.uk

Abstract

Approximate methods of analysis are essential tools at the preliminary design phase of any building project. For multi-story multi-bay lateral-force-swayed frames, the available classical approximate methods are the Portal and the Cantilever techniques. It is known that the Portal method gives more accurate results for low to medium rise frames. This investigation showed that it is possible to obtain more accurate analysis results than those availed by the Classical Portal method, if adequate account is taken of the essential factors that affect members' inflection points and the shear force resisted by the columns. The research method consisted of a study of frame behaviour through a detailed examination of the analysis results of ninety multi-story fixed-feet sway frame variants analysed with the aid of computer software, SAP2000. Interior-to-exterior shear ratios for equal and unequal bay frames, as well as column inflection points were obtained to serve as practical aids for preliminary analysis/design of fixed-feet *multistory sway frames. Equal and unequal bay five story frames were analysed to show the validity of the recommended design parameters.*

Keywords: Approximate, Preliminary Design, Portal Method, Sway Frames, Shear Ratio, Stiffness Ratio, Inflection Point.

Introduction

Determination of stress resultants due to lateral loads is an essential part of the analysis/design of multi-story reinforced concrete building frames. Such frames are highly redundant and their manual analysis often presents computational challenges. Fortunately, such hyper-static rigidly jointed sway frames can today be efficiently analysed by digital computation on the basis of the concise and systematic approach embodied in the displacement method in its matrix formulation.

However, simplified approximate methods adapted to hand calculation are still valuable for preliminary design analysis and to check computer results based on exact analysis methods. The preliminary design stage is particularly important because the future of any project (including overall cost) greatly depends on the quality of conceptual design (Olotuah and Arum, 2007). The importance of the conceptual/preliminary design stage in the entire design process was aptly captured by Mola (2007), who said that at present time conceptual design has become a well-defined way of approaching structural problems and many designers conceive their projects according to its basic principles.

On the role of approximate methods, Nilson, Darwin and Dolan (2004) wrote that in spite of the development of refined methods for the analysis of beams and frames, increasing attention is being paid to various approximate methods of analysis. This is because prior to performing a complete analysis of an indeterminate structure, it is necessary to estimate the proportions of its members to determine their relative stiffness, upon which the analysis depends. On the other hand, the dimensions can be obtained on the basis of approximate analysis. This fact was collaborated by Leet and Uang (2005) who affirmed that if designers understand the behaviour of a particular structure, they can often use an approximate analysis to estimate closely, with a few simple computations, the approximate magnitude of the forces at various points in the structure. Furthermore, they maintained that designers use the result of an approximate analysis to size the main members of a structure during the preliminary design phase and to verify the accuracy of an exact analysis.

Unfortunately, the classical approximate methods are often applied in their raw theoretical formulations. An understanding of frame behaviour can help the analyst/designer adjust the assumptions that form the basis of the approximate methods, to suit particular situations. The objectives of the present work were therefore to examine the major approximate analysis methods for lateral-load-swayed reinforced concrete open frames and using the Portal method as a case study, to quantitatively show how analysis results can be improved by effecting some adjustments to the theoretical format, based on the understanding of both the basis for the theoretical formulation, and frame behaviour.

Approximate Methods of Analysis

Various techniques are available for approximate analysis of continuous beams and rigid frames for gravity load. Notable among the techniques are those of guessing the location of points of inflection and estimating the values of the member-end moments. Approximate methods are especially necessary for the analysis of highly redundant multi-story reinforced concrete moment-resisting systems subjected to lateral forces. The two established classical approximate methods for such frames are the Portal and the Cantilever methods (Wang, 1983; Kong, et. al., 1983; Kassimali, 1993; Englekirk, 2003; Raju, 2005). Approximate analysis using these methods is usually performed by effectively reducing the degree of static indeterminacy by suitable moment releases. Recognition of the major mode of racking deformation of the frame makes possible realistic predictions of the resulting points of contra-flexure in both beams and columns. The degree of indeterminacy is reduced by the number of inflection points assumed. The Portal and Cantilever methods make use of the same assumption that points of contra-flexure occur at the mid-height positions of all columns and at the mid-span positions of all beams. For single-bay frames, this single assumption is sufficient to reduce the structure to a statically determinate system. For multi-bay frames however, additional assumptions are made. The additional assumptions account for the difference in the two techniques. For the Portal method, the additional assumption is that the shear force resisted by an interior column is twice that by an exterior column. In the Cantilever method, the underlying idea is the recognition of the bending action of the entire frame as the dominant one compared to the panel shearing action. As a consequence, the additional assumption in the method is that since the columns act effectively as the fibres in a beam, the axial stress in each column is proportional to its distance from the centroidal axis, or centre of gravity, of the column areas as a whole.

Another valuable if relatively less common approximate method for multi-story lateral-load-swayed frames is the D-value method (Muto, 1974). The D-value method consists essentially in expressing frame rigidity and distribution of lateral forces in terms of a distribution coefficient termed "D-value", which was based mainly on beam-to-column stiffness ratio and on the column stiffness. This method is particularly useful for the manual analysis/design of earthquake-resistant structures because it can be used to account for the interaction between frames and other lateral-load-resisting assemblies such as shear walls in the complete building. It was one of the most effective methods of analyzing earthquake-resistant buildings in the pre-computer era. The method rigorously considers the effects of the stiffness ratios of the bounding beams, beam-to-column stiffness ratios, and the stiffness ratios of the columns, on the shear-resisting capacity of a column. Thus, unlike the Portal and the Cantilever methods, the D-value method assumes an almost accurate knowledge of members' relative dimensions. Unfortunately, at the preliminary analysis stage, there is only approximate knowledge of members' relative dimensions. Therefore considering the relative rigour involved and the fact that the computer is now available for the final analysis/design phase, the D-value method may not be very attractive today as a preliminary analysis tool, especially for open frames.

From the foregoing, it can be inferred that the Portal and the Cantilever methods still remain the choice techniques for preliminary analysis of sway moment-resisting systems. However, Arum and Aderinlewo (2005, 2006) have used quantitative parameters to show that in low to medium-rise building frames, the analysis results obtained using the Portal method are closer to the exact results than do the results by the Cantilever method. This work is intended to show how the application of knowledge of frame behaviour to the fundamental assumptions in the theory of the Portal method can be used by the design engineer to obtain more accurate values of analysis actions at the preliminary analysis/design phase.

Portal Method

Theoretical Basis and Possible Sources of Inaccuracy

As mentioned earlier, one of the two assumptions that form the basis of this method is that points of contra-flexure occur at the mid-height positions of all columns and at the mid-span positions of all beams. Usually beams are designed to be stiffer than the columns because of the relatively high magnitude of gravity loading they must resist. It is also known from frame behaviour that for laterally loaded portal frames, when beam-to-column stiffness is very high, neglecting axial deformation, the beam deflects horizontally as a rigid body and, for compatibility, the columns bend in double curvature and the points of contra-flexure in both columns and beams tend to be near their mid-lengths. In practice however, although the beams are usually stiffer than the columns, they are not infinitely rigid and errors naturally arise in the analysis results due to this fact. In addition the column end conditions are usually not the same throughout a building. The stiffness of the upper beams for instance may be different from that of the lower beams. In addition, the ground floor columns are bounded at the lower end by the foundation or foundation beams and therefore the stiffness at the lower boundary depends on whether the frame is assumed fixed or pinned at the bottom. A column fixed at the base will have its inflection point substantially moved upwards from its mid-length especially if the upper beam-to-column stiffness ratio is not great. On the other hand, if the column is pinned at the base, inflection point will not occur at all within the length of the column since a pin cannot transmit moment. Furthermore, it has been well-established from studies of frame behaviour (Muto, 1974; Wang, 1983; Arum and Aderinlewo, 2005) that at the topmost story of any multi-story building, the effect of the stiffnesses of the top and bottom bounding beams is to move the inflection point of the column downwards from its mid-length. Since the assumption of inflection points at column and beam mid-lengths ignores the influence of the various aforementioned factors, inaccuracies arise in the analysis results, which are usually significant for the topmost and bottommost floors.

The second assumption of the Portal method is that the shear force borne by an interior column is twice that of an exterior column. This assumption was based on the fact that gravity load constitutes the primary loading in frames and since the tributary load area for an interior column is often about twice that for the exterior columns, the interior column usually has greater cross-sectional dimensions. In recognition of the shearing panel actions across the panels as the dominant behaviour mode of the frame and since the shear distributed to the columns supporting a particular floor is approximately proportional to their flexural stiffness (EI/h), the Portal method therefore assumes the shear resisted by the interior column to be twice that by the external column. In practice however, although the cross-sectional dimensions of the interior column is usually greater than those of the exterior column, the stiffness is rarely as much as twice.

Buildings for which the exterior walls are constructed from heavy masonry and the tributary floor areas on columns are not great (small slab panel areas) often have about the same cross-sectional dimensions and therefore about the same magnitude of bending stiffness for the interior and the exterior columns. In other buildings the ratio of the interior-to-exterior column stiffness may be about 1.5. For such cases, errors (which can be significant) are introduced in the analysis based on the assumption that the stiffness ratio is equal to two (2). Such errors are even heightened if at a particular floor level, the bays are of different widths while the adjacent beams are of the same cross-section. In such a case the shorter beam is stiffer and will hold down the exterior column to its side more rigidly than the longer less rigid beam will do for the exterior column to its own side. Consequently, even if the interior column cross-sectional dimensions are twice those of the exterior columns, the column to the side of the stiffer beam will still resist greater shear than the one to the side of the more flexible beam. In such a case the proportion of the shear resisted by the interior column will tend to decrease with respect to the shear resisted by the more rigidly held of the exterior columns whereas it will increase with respect to the shear borne by the less rigidly bounded of the exterior columns.

Measures for Improving Accuracy of Analysis Results

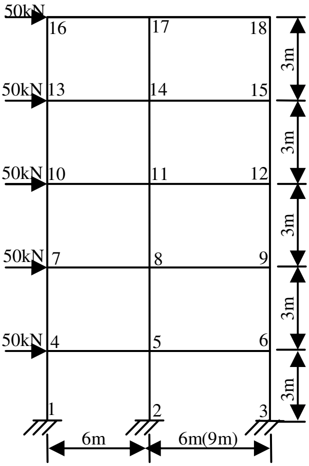
Having considered the theoretical basis for the assumptions made in the Portal method, as well as the various factors that impact on the accuracy of the analysis, the possibility of improving the accuracy of the analysis can now be better appreciated. Thus, based on the engineer’s understanding of frame behaviour and the various factors that affect members’ inflection points and column’s ability to resist shear, including an estimated ratio of interior-to-exterior column second moments of area, more accurate values for inflection point positions and the magnitude of shear resisted by each column can be obtained. The column end moments are then easily computed by multiplying column shears by the appropriate distances from the inflection point. Beam end moments are determined by distributing the column moment at each joint to the beams in proportion to the later's stiffnesses.

Methodology and Theoretical Analysis

Basis for Proposed Analysis/Design Aids

In this investigation ninety fixed-feet multi-story reinforced concrete frame variants were analysed using commercial general-purpose computer software, SAP2000, which is based on the stiffness method (an exact method). Forty five of the variants had two equal bays of 6m each. Each of the other forty five had two unequal bays of 6m and 9m. A nodal lateral force of 50kN was applied at each floor level for each frame. The analytical model used is shown in Fig.1 for the case of a 5-story frame.

Figure 1: Frame Analytical Model



The frames were analysed for varying interior-to-exterior column stiffness ratios, beam-to-exterior column stiffness ratios and beam stiffness ratios. By employing simple algebraic expressions, the column inflection points were determined. Also obtained were the proportions of the shears resisted by the interior and the exterior columns.

Validation of Results Obtained Using Adjusted Portal Method

Two 5-story fixed-feet equal and unequal-bay frames were analysed by the Portal method, first, strictly according to its theoretical format, namely, location of the inflection points at mid-lengths of columns and beams, and the interior column shear being twice that of each exterior column. Next, the Portal method was employed in its modified format, taking cognizance of the positions of the inflection points as well as the interior-to-exterior column shear ratio obtained through the study of the results of the computer-aided (SAP) exact analysis, as described in the previous section. The results of the Portal method and those of the judgement-based Portal method (which henceforth will be loosely termed "modified Portal method") were compared with the results of exact analysis.

Results and Discussion

This investigation revealed that the average ratio of the interior-to-exterior column shear is as shown in Table1, for equal bay multi-story frames with the number of stories not greater than fifteen.

Table 1: Recommended Values of Interior-to-Exterior Column Shear Ratios for Equal Bay Fixed-Feet Frames, for Different Members' Stiffness Ratios

Bay Widths: $B_1 = B_2 = 6m$				
Story	$I_{ic} = I_{ec}$		$I_{ic} = 2I_{ec}$	
	$I_b = 2I_c$	$I_b = 3I_c$	$I_b = 2I_c$	$I_b = 3I_c$
First	1.30	1.28	2.0	2.0
Intermediate	1.73	1.65	2.0	2.0
Last	1.91	1.75	2.0	2.0

For frames with unequal bay dimensions, average ratios of the exterior column shears as fractions of the interior column shear is as presented in Table 2.

Table 2: Recommended Values of Exterior Column Shear as a Fraction of Interior Column Shear for Unequal Bay Fixed-Feet Frames, for Different Members' Stiffness Ratios

Bay Widths: $B_1 = 6m$; $B_2 = 9m$								
Story	$I_{ic} = I_{ec}$				$I_{ic} = 2I_{ec}$			
	$I_b = 2I_c$		$I_b = 3I_c$		$I_b = 2I_c$		$I_b = 3I_c$	
	RBSR = 1	LRBSR = 0.67	RBSR = 1	LRBSR = 0.67	RBSR = 1	LRBSR = 0.67	RBSR = 1	LRBSR = 0.67
First	0.81	0.71	0.82	0.73	0.54	0.46	0.54	0.47
Intermediate	0.64	0.50	0.66	0.53	0.58	0.44	0.58	0.44
Last	0.55	0.46	0.57	0.53	0.58	0.44	0.55	0.47

Note. RBSR = reference beam stiffness ratio;
LRBSR = less rigid beam stiffness ratio.

Table 3 shows the average positions of column inflection points as fractions of the column height, measured from column lower end.

Table 3 Recommended Values of Column Inflection Points as a Fraction of Column Height, for Different Stiffness Ratios of Frame Members

Story	$I_{ic} = I_{ec}$				$I_{ic} = 2I_{ec}$			
	$I_b = 2I_c$		$I_b = 3I_c$		$I_b = 2I_c$		$I_b = 3I_c$	
	Exterior Column	Interior Column	Exterior Column	Interior Column	Exterior Column	Interior Column	Exterior Column	Interior Column
First	0.64	0.56	0.60	0.54	0.63	0.63	0.59	0.59
Second	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
Intermediate	0.44	0.48	0.46	0.49	0.45	0.45	0.47	0.47
Last	0.33	0.43	0.38	0.46	0.35	0.35	0.40	0.40

Note. Inflection points are measured from column lower end

The column end moments obtained from the sample analyses for 5-story fixed-feet frame using the Portal method, the proposed Modified Portal method and exact analysis, are presented in Tables 4 and 5 respectively for equal and unequal bay frames.

Table 4 Column End Moments (kNm) in Equal Bay Fixed-Feet 5-Story Frame for Different Members' Stiffness Ratios using Different Methods of Analysis

Bay Widths: $B_1 = B_2 = 6$m									
Column End Moment M_{ij}	Portal	$I_{ic} = I_{ec}$				$I_{ic} = 2I_{ec}$			
		$I_b = 2I_c$		$I_b = 3I_c$		$I_b = 2I_c$		$I_b = 3I_c$	
		Modified Portal	Exact	Modified Portal	Exact	Modified Portal	Exact	Modified Portal	Exact
M_{14}	93.75	145.46	145.51	137.20	138.14	118.13	119.09	110.63	111.64
M_{41}	93.75	81.82	82.77	91.46	91.48	69.38	69.88	76.88	76.89
M_{47}	75.00	80.43	75.74	82.19	77.85	75.00	72.55	75.00	72.84
M_{74}	75.00	80.43	84.41	82.19	85.53	75.00	77.05	75.00	76.92
$M_{7,10}$	56.25	50.08	53.43	56.72	56.73	50.63	50.63	52.88	52.39
$M_{10,7}$	56.25	67.55	67.42	66.58	66.80	61.88	61.93	59.63	60.14
$M_{10,13}$	37.50	35.39	32.67	37.81	35.76	33.75	31.39	35.25	33.39
$M_{13,10}$	37.50	45.04	47.76	44.39	46.48	41.25	43.60	39.75	41.60
$M_{13,16}$	18.75	12.66	12.82	15.20	15.28	13.13	13.25	15.00	14.93
$M_{16,13}$	18.75	25.71	25.57	24.79	24.68	24.38	24.25	22.50	22.56
M_{25}	187.5	165.45	166.95	158.05	158.17	236.25	236.19	221.25	221.92
M_{52}	187.5	129.99	128.86	134.63	134.28	138.75	138.25	153.75	152.69
M_{58}	150.0	139.14	138.60	135.62	135.83	150.0	145.86	150.0	146.13
M_{85}	150.0	139.14	140.48	135.62	137.03	150.0	154.33	150.0	153.99
$M_{8,11}$	112.5	100.18	95.50	99.68	98.63	101.25	101.15	105.75	104.73
$M_{11,8}$	112.5	108.53	108.86	103.75	104.36	123.75	123.82	119.25	120.25
$M_{11,14}$	75.00	66.79	64.83	66.46	64.89	67.50	62.80	70.50	66.80
$M_{14,11}$	75.00	72.35	74.30	69.17	70.62	82.50	87.20	79.50	83.21
$M_{14,17}$	37.50	31.50	31.64	32.20	31.96	26.25	26.50	30.00	29.87
$M_{17,14}$	37.50	41.76	41.57	37.79	38.13	48.75	48.50	45.00	45.13

Table 5 Column End Moments (kNm) in Unequal-Bay Fixed-Feet 5-Story Frame for Different Analysis Methods and Members' Stiffness Ratios

Bay Widths: $B_1 = 6$m; $B_2 = 9$m									
Column End Moment M_{ij}	Portal	$I_{ic} = I_{ec}$				$I_{ic} = 2I_{ec}$			
		$I_b = 2I_c$		$I_b = 3I_c$		$I_b = 2I_c$		$I_b = 3I_c$	
		Modified Portal	Exact	Modified Portal	Exact	Modified Portal	Exact	Modified Portal	Exact
M_{14}	93.75	154.29	154.19	144.70	145.18	127.58	127.07	118.87	118.22
M_{41}	93.75	86.79	87.05	96.47	95.89	74.93	73.90	82.61	81.25
M_{47}	75.00	89.72	86.69	90.41	87.35	86.15	83.19	86.15	82.29
M_{74}	75.00	89.72	94.12	90.41	94.19	86.15	86.53	86.15	85.60
$M_{7,10}$	56.25	59.22	59.99	62.39	62.32	58.14	57.49	60.73	58.38
$M_{10,7}$	56.25	75.36	75.15	73.24	73.09	71.07	69.85	68.48	66.77
$M_{10,13}$	37.50	39.48	36.20	41.59	38.64	38.76	35.31	40.48	36.73
$M_{13,10}$	37.50	50.25	52.77	48.83	50.22	47.37	48.86	45.65	45.72
$M_{13,16}$	18.75	13.54	13.67	15.47	15.56	15.08	14.69	16.33	15.83
$M_{16,13}$	18.75	27.50	27.35	25.24	25.33	28.00	26.81	24.50	23.96
M_{25}	187.5	166.67	171.89	158.82	162.25	236.25	244.54	220.15	228.18
M_{52}	187.5	130.96	125.36	135.30	132.53	138.75	130.46	152.99	147.04
M_{58}	150.0	140.19	139.65	136.98	137.52	148.52	146.23	148.52	146.16
M_{85}	150.0	140.19	141.76	136.98	139.10	148.52	154.46	148.52	154.64
$M_{8,11}$	112.5	100.93	99.63	100.68	99.38	100.25	99.32	104.71	103.42
$M_{11,8}$	112.5	109.34	111.12	104.79	106.63	122.53	126.04	118.07	122.03
$M_{11,14}$	75.00	67.29	64.47	67.12	65.11	66.83	60.56	69.80	65.26
$M_{14,11}$	75.00	72.90	76.22	69.86	72.38	81.68	89.68	78.71	85.02
$M_{14,17}$	37.50	32.10	31.28	32.86	31.90	25.99	24.62	29.70	28.48
$M_{17,14}$	37.50	42.54	43.45	38.57	39.61	48.26	50.49	44.55	46.57
M_{36}	93.75	135.24	143.84	128.83	135.99	108.68	117.67	103.47	109.96
M_{63}	93.75	76.08	67.67	85.88	78.16	63.83	56.36	71.91	65.35
M_{69}	75.00	70.10	63.97	72.60	66.13	65.34	62.22	65.34	62.94
M_{96}	75.00	70.10	73.81	72.60	75.70	65.34	67.36	65.34	68.36
$M_{9,12}$	56.25	46.27	43.36	50.09	47.86	44.10	41.46	46.06	44.56
$M_{12,9}$	56.25	58.88	60.74	58.81	60.71	53.91	55.84	51.95	54.84
$M_{12,15}$	37.50	30.84	25.78	33.40	30.02	29.40	24.97	30.71	28.21
$M_{15,12}$	37.50	39.24	44.56	39.20	43.62	35.94	40.61	34.63	39.05
$M_{15,18}$	18.75	11.33	9.28	14.39	12.78	11.43	9.81	13.96	12.55
$M_{18,15}$	18.75	22.99	24.98	23.47	24.81	21.24	23.59	20.93	12.60

From the results shown in Tables 1 through 5, it can be easily appreciated that the values for the column end moments significantly depend on the interior-to-exterior frame stiffness ratios, on the stiffness ratios of the beams at the column boundaries, and on the story being considered. However, these moments are not appreciably sensitive to the beam-to-column stiffness ratios. Tables 4 and 5 indicate that the moments obtained by the modified Portal method (by employing Tables 1 through 3) are for practical purposes, essentially the same as those obtained by exact analysis. The above is true both for equal and unequal-bay frames. On the other hand, substantial variations exist between the end moments obtained by the Portal method (in its strict theoretical formulation) and those obtained using exact analysis.

Conclusion

The following conclusions follow from this study:

1. Use of the Portal method in its strict theoretical format can lead to inaccuracies which will result in increased preliminary analysis time and hence project delay.
2. Tables 1 through 3 can serve as practical design aids at the feasibility/preliminary design phase of a sway multi-storey building project.

Notations

B_1, B_2	bays;
I_b	beam second moment of area;
I_c	column second moment of area;
I_{ec}	exterior column second moment of area;
I_{ic}	interior column second moment of area;
LRBSR	less rigid beam stiffness ratio;
M_{ij}	bending moment at end i of member ij;
RBSR	reference beam stiffness ratio.

References

Arum C. and Aderinlewo O.O. (2005). Comparison of Cantilever and Portal Methods with Elastic Analysis for Building Frames. *International Research Journal in Engineering, Science and Technology*, Vol. 2, No.2, pp176-186.

Arum C. and Aderinlewo O.O. (2006). Inflection Points of Wind-swayed Reinforced Concrete Frames. *Journal of Engineering and Engineering Technology* (FUTAJEET), Federal University of Technology, Akure, Vol.5, No.1, pp46-51.

Coull A. and Smith B.S. (1983). Tall buildings, in Kong F.K., Evans R.H., Cohen E. and Roll F. (eds): *Handbook of Structural Concrete*, McGraw-Hill Book Company, UK.

Englekirk R. (2003). *Steel Structures. Controlling Behaviour through Design*. John Wiley & Sons, Singapore.

Kassimali A. (1993). *Structural Analysis*. PWS-Kent Publishing Co., London.

Leet K.M. and Uang C. (2005). *Fundamentals of Structural Analysis*. McGraw-Hill, New York.

Mola F. (2007). Introductory remarks and conference aims, 4th International Specialty Conference on *The Conceptual Approach to Structural Design*, Venice, Italy.

Muto K. (1974). *Aseismic Design Analysis of Buildings*. Maruzen Company Ltd., Tokyo.

Nilson A.H., Darwin D. and Dolan C.W. (2004). *Design of Concrete Structures*. McGraw-Hill, New York.

Olotuah A.O. and Arum C. (2007). The Nexus of Intuition and Knowledge of Structures in Architecture. *Research and Development Online Resources of the Construction Industry Institute*, Hong Kong http://www.ciihk.org.hk/rdtest/en/download_search_result.php.

Raju N.K. (2005). *Advanced Reinforced Concrete Design*. CBS Publishers, Delhi.

Wang C.K. (1983). *Intermediate Structural Analysis*. McGraw-Hill, New York.