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Application of Monte Carlo Method to Steady State Heat Conduction Problems

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A. O. Oluwajobi -Department of Mechanical Engineering, Obafemi Awolowo University, Ile-Ife. jide_ogunjobi@yahoo.com

A. B. Jeje - Department of Mechanical Engineering, University of Ibadan, Ibadan – Nigeria

Abstract

The Monte Carlo method was used in modelling steady state heat conduction problems. The method uses the fixed and the floating random walks to determine temperature in the domain of the definition of the heat conduction equation, at a single point directly. A heat conduction problem with an irregular shaped geometry and prescribed boundary temperatures was analyzed. Computer programs for the analysis were coded using VAX FORTRAN on a VAX 4.4 mainframe. The fixed random walk and the floating random walk methods gave comparable results of 507.26^{0}K and 506.98^{0}K respectively for temperature at a point. The Monte Carlo method also gave a result of 50.58^{0}C, similar to that of 50.00^{0}C for a particular node, with the Finite Difference method (using Gauss- Seidel iteration) for a 2-dimensional regular shaped heat conduction problem.

Keywords: *Monte Carlo Method, Fixed Random Walk, Floating Random Walk, Modelling, Steady-State Heat Conduction*

Introduction

The conduction of heat in solids has numerous applications in various branches of science and engineering. So, there is considerable interest in the solution of heat conduction problems with the ultimate objective of obtaining useful and practical information. Up to date, there are various methods that are available for the solution of these problems, ranging from exact to approximate numerical techniques. Among the numerical methods are the Finite Difference Method, the Finite Element Method, the Boundary Element Method, and the Monte Carlo Method. When using many of these techniques, all mesh point temperatures must be determined simultaneously, whereas the Monte Carlo method is able to solve for the temperature at any isolated point in the required domain.

The Monte Carlo technique is basically any technique making use of random numbers to solve a problem. It is characteristically used to describe probability sampling techniques that approximate the solution of mathematical or physical problems. Problems handled by the Monte Carlo method are of two types. The first is called probabilistic and the second type is called deterministic, the naming depending on the behaviour and outcome of random processes.

Whether or not the Monte Carlo method can be applied to a given problem doesn't depend on the stochastic nature of the system being studied, but only on the investigator's ability to formulate the problem in such a way that random numbers may be used to obtain the solution. The Monte Carlo method can be applied wherever it is possible to establish equivalence between the desired result and expected behaviour of a stochastic system. In this study, a 2-dimensional arbitrary shaped body with prescribed temperatures at the boundaries was modelled for temperature distribution.

Literature Review

It had been known since early last century that probability-sampling (Monte Carlo) techniques could be employed in solving partial differential equations. Lord Rayleigh in 1899 demonstrated the relationship that exists between stochastic processes and parabolic differential equations. Also, Courant and his co-workers established a similar relationship for elliptic differential equations in 1928. Todd (1954) further worked with Laplace equation in a square geometry having prescribed boundary temperatures.

The Monte Carlo technique was employed by Howell and Perlmulter (1964) to solve radiation heat transfer problems. They concluded that the method is a simple and accurate way of solving radiative heat problems. They stressed further that the technique is especially flexible for use with complex configuration and can readily include such effects as scattering.

Haji-Sheikh and Sparrow (1967) applied Monte Carlo methods to thermal conduction problems, and they motivated research on floating point random walks. Other researchers also made contributions to the field. Chandler et al (1968) worked on the solution of steady state convection problems using the fixed random walk method. Burmeister (1965) devised a floating random walk method for 2-D steady state conduction problems with straight convecting boundaries. Grigogriu (2000) proposed a different Monte Carlo method based on the properties of Brownian motion and Ito processes. The new method is based on a theoretical formulation different from the conventional method. Deng and Liu (2002) developed a Monte Carlo algorithm for solving 3-D heat transfer problems in biological tissues with large blood vessels. Chatterjee (2006) used a new 3-D floating random walk algorithm to solve a nonlinear Poisson-Boltzmann (NPB) equation. Now, interest is on parallelized algorithms, where the advantage is that each random walk is independent, so that the computational procedure is inherently parallelizable and an almost linear increase in computational speed is expected with increase in the number of processors.

Theoretical Background and Methodology

Truly Random Numbers

A sequence of truly random numbers is unpredictable and therefore is irreproducible. Such a sequence can only be generated by a random physical process e.g. radioactive decay, thermal noise in electronic devices, cosmic ray arrival times, et cetera. However in Monte Carlo studies, one often uses the word 'random' with a slightly different meaning. It is usually applied to sequences of which once they have been determined, are not at all random in the statistical sense, but may have some properties, which are similar to the properties of a random sequence. They are sometimes called pseudo random numbers.

Random Walks

A basic ingredient of the Monte Carlo solution of differential equations is the random walk. In the conventional random walk, the length of the step and the pathways are fixed in advance. This type of random walk is known as a fixed random walk. In contrast to this, there is a more flexible random walk, which possesses the property that neither the length of a step nor the mesh points are pre-selected. This is known as the floating walk.

The fixed walk technique has long computational times and has difficulties with complex boundary conditions at irregular boundaries. But, the floating walk procedure is relatively indifferent to the shape of the boundary or the boundary conditions, and the computational time is substantially less.

The Fixed Random Walk

Consider an arbitrary shaped 2-dimensional solid with a grid of mesh size l x l superimposed on it (Figure1).

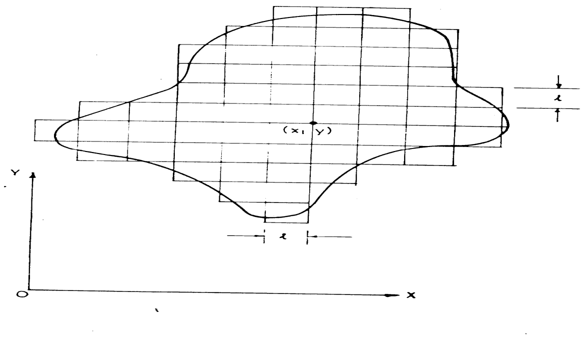

Figure 1 . 2-Dimensional region of arbitrarily shaped body with superposed grid.

Suppose the temperature at an interior point is desired. The problem is governed by the familiar Laplace equation.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \qquad (1)$$

It is already established that the temperature at any mesh point T(x,y), by finite difference, is expressible in terms of the temperatures at the surrounding mesh points as follows

$$T(x, y) = p_{x+}T(x+l, y) + p_{x-}(x-l, y) + p_{y+}T(x, y+l) + p_{y-}T(x, y-l)$$

$$where\ p_{x+} = p_{x-} = p_{y+} = p_{y-} = \frac{1}{4} \qquad (2)$$

Equation (2) may be given the probability interpretation viz; that if a random-walking particle is instantaneously at the point (x, y), it has an equal probability of stepping to any one of the points (x+l, y), (x-l,y), (x,y+l), (x,y-l). The probability is given by the quantities p_{x+}, p_{x-}, p_{y+}, p_{y-}, which is equal to ¼.

A digital computer may then be programmed to supply a random number between 0 and 1. If the random number lies between 0.00 and 0.25, the particle is instructed to move from (x,y) to (x+l,y); if the random number lies between 0.25 and 0.50, the particle walks from (x,y) to (x-l,y); if the random number is between 0.50 and 0.75, the particle walks from (x,y) to (x,y+l); and if the random number lies between 0.75 and 1.00, the particle walks from (x,y) to (x,y-l).

The computation of the temperature at any interior mesh point (x, y) as shown in the Figure 1 is as follows. Assume that the boundary temperature, T_w, is a prescribed function of the point along the boundary. The particle then starts to wander from point to point on the nodes of the grid. This continues until it reaches a mesh point on the boundary, then the walk is terminated and the known temperature at that boundary point is tallied. See Figure (2)

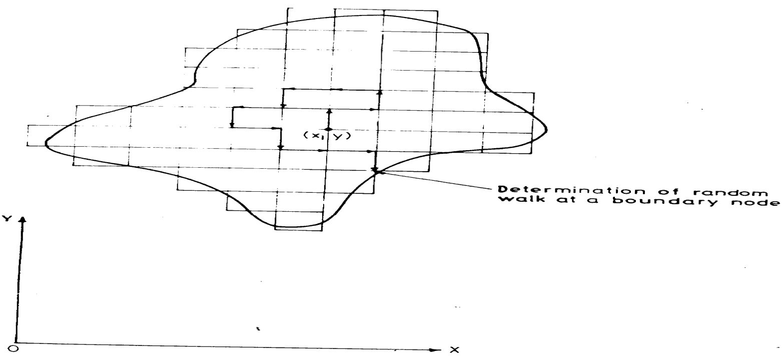

Figure 2 · A Typical random walk.

Let the temperature at the end of the first walk be designated $T_w(1)$. Then a second particle released from the departure point (x,y) proceeds in a similar manner, to wander among the grid points until it reaches a boundary point, where the walk is terminated and corresponding boundary temperature $T_w(2)$ is tallied. This same procedure is repeated for the next particle, up to the Nth particle released from (x,y), (where N is the maximum number of walk allowed), and the corresponding temperatures are recorded.

The Monte Carlo solution for T(x,y) is expressed as

$$T(x,y) = \frac{1}{N}\sum_{i=1}^{N} T_w(i) \qquad (3)$$

This temperature obtained is identical to that obtained by an exact solution of the finite difference representation in the limit as N tends to infinity $(N \rightarrow \infty)$

If there is a volume heat source within the body, the finite difference representation for T(x,y) becomes

$$T(x,y) = p_{x+}T(x+l,y) + p_{x-}(x-l,y) + p_{y+}T(x,y+l) + p_{y-}T(x,y-l) + \frac{Sl^2}{4k} \qquad (4)$$

where S is the rate of heat generation per unit volume, k is the thermal conductivity and l is the mesh size.

Equation (4) has the same probabilistic interpretation as equation (2). However, the term $\frac{Sl^2}{4k}$ in equation (4) must be scored for each step of a random walk. So if m_i steps are required for the ith random-walking particle originating at (x,y) to reach the boundary, and T_w(i) is the boundary temperature at the terminal point of the walk, then, $T_w(i) = m_i \frac{Sl^2}{4k}$ is scored for each of N random walks originating from (x,y). The

Monte Carlo result for T(x,y) with internal heat generation source is

$$T(x,y)=\frac{1}{N}\sum_{i=1}^{N}T_w(i)+[\frac{Sl^2}{4k}]\frac{1}{N}\sum_{i=1}^{N}m_i \qquad (5)$$

As earlier mentioned, the fixed-random walk Monte Carlo has difficulty when used with irregular shaped bodies having boundary conditions that involve normal derivatives. This is because the grid lines comprising the finite-difference mesh do not meet the boundary at right angles. These difficulties are removed by the use of floating point random walk.

The Floating Random Walk

This is a type of walk wherein neither the step size nor the mesh points are pre-assigned. The basis of the floating random walk may be deduced by considering the exact temperature solution for the homogeneous 2-dimensional region with radius r and centre at (x,y). The temperature at (x,y) can be represented with approximation in terms of the temperature distribution $T(r,\omega)$ on the boundary and

$$T(x,y)=\int_0^1 T(r,\omega)dF(\omega) \qquad (6)$$

and $F=\frac{\omega}{2\pi}$

where ω is the angular coordinate on the circle.

The quantity F is the probability distribution function. And since $\frac{dF}{d\omega}=\frac{1}{2\pi}$ is a constant, it shows that for an elementary arc length $d\omega$, the probability dF is independent of ω.

Therefore a random-walking particle instantaneously situated at the centre of the circle and preparing for its next step has equal probability of being located at any angular position ω on the circumference.

Suppose that a random-walking particle is at some point (x_i, y_i) after executing i prior steps. The next step (i.e. $i+l$ step) is executed as follows. A circle is constructed with (x_i, y_i) as the centre and with radius, r, which is equal to the shortest distance between (x_i, y_i) and the boundary. Then a random number between 0 and 1 is obtained, which when multiplied by 2π gives the circumferential location ω_i to which the particle moves. The new coordinates are denoted as (x_{i+l}, y_{i+l}). Geometrically, the above procedure is illustrated in Figures (3) and (4).

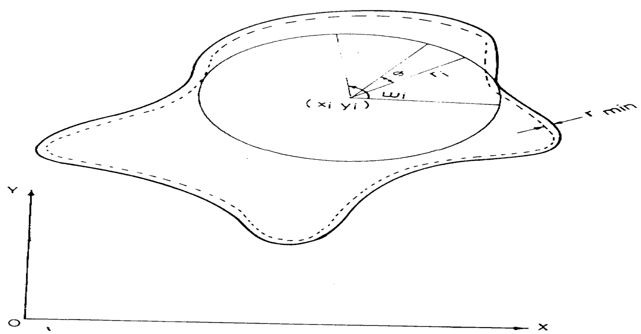

Figure 3: Schematic representation for a 2-Dimensional floating random walk.

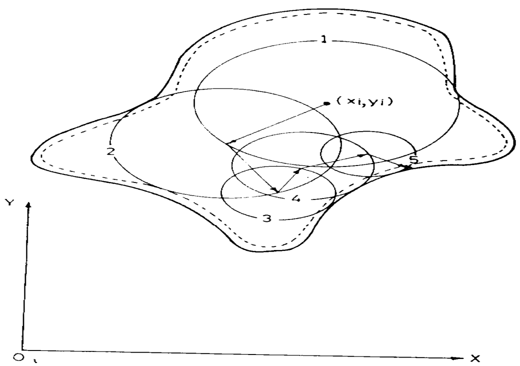

Figure 4 · A Typical floating walk

The coordinates x_{i+l} *and* y_{i+l} are expressed as

$$x_{i+l} = x_i + r_i \cos\omega_i \;, \qquad y_{i+l} = y_i + r_i \sin\omega_i \qquad (7)$$

The next step of the random walk is carried out by using (x_{i+l}, y_{i+l}) as the centre and constructing a circle whose radius r is the short distance between this centre and the boundary and et cetera.
Consider a 2-dimensional solid with prescribed boundary temperature T and no internal heat generation. To determine the temperature at any interior point (x,y), random walking particles are successively set into motion from (x,y).

Each particle wanders throughout the body until it reaches the boundary (or an approximate representation of the boundary), where the walk terminates and the corresponding boundary temperature is tallied. If there are N particles, then T(x,y) is given by equation (4). See Figure (4).

When there is an internal heat generation, $\frac{Sr_j^2}{4k}$ must be scored at each step of each random walk. Suppose that in its $(i+l)$ step, the jth random-walking particle moves a distance r_{ji}. If m_i represents the number of steps executed during the ith walk, the temperature

$$T(x,y) = \frac{1}{N}\sum_{i=1}^{N} T_w(i) + \frac{1}{N}\sum_{i=1}^{N}[\sum_{j=0}^{M_{i-1}} \frac{Sr_{ji}^2}{4k}] \qquad (8)$$

Boundary Representations

When a random-walking particle approaches a boundary (for floating walk), the radius r of the circle defining the locus of the next step may become very small. This would in turn increase the time required to carry out practical calculations.

In order to avoid this state, it is wise to employ any one of the following approximate representations of the boundary.

(i) When the radius is smaller than a preassigned value $r_{\min}$, the floating random walk can be regarded as being at the nearest point on the boundary.

(ii) If a random-walking particle moves to an angular position that deviates by less than a preassigned quantity $\pm\Theta$ from the position where the circle contacts the boundary, then the random walk can be regarded as having arrived at the boundary.

(iii) When the radius r_i is smaller than a pre assigned value $r_{\min}$, the finite difference form of the heat conduction equation can be employed to establish probabilities for the next step of the walk.

(iv) When the radius r_i is smaller than a pre assigned value $r_{\min}$, the solution of the 1-dimensional energy equation (applied normal to the boundary) can be employed to establish the probabilities for the next step of the walk.

The first approximate representation, (i) was used for the computation in this study. This creates a strip adjacent to the boundary as shown in figures (3) and (4) and within which the floating random walk is treated specially. The $r_{\min}$ was chosen to be equal to ¼, where l is the step size of an admissible finite-difference grid.

Boundary Conditions

The differential equation encountered in heat conduction problems will have numerous solutions unless a set of boundary condition and/or initial conditions (for transient problems) are prescribed. The boundary conditions specify the temperature or the heat flow at the boundaries of the region and the initial condition specifies the temperature distribution in the medium at the origin of the time coordinate.

Consider, when a random-walking particle arrives at a boundary having prescribed temperature, the walk is terminated. Such a boundary is known as an absorbing barrier. Now, suppose the heat flux is prescribed at the boundary, that is

$$\frac{\partial T}{\partial n} = -\frac{q}{k} \qquad (9)$$

where n is the direction of positive heat flow and q is the heat passing into the body per unit time and area. The Taylor's series expansion of the derivative, retaining terms up to the second order gives

$$T_w = T_{\Delta n} + (\frac{q}{k})\Delta n + (\frac{S}{2k})\Delta n^2 \qquad (10)$$

where $T_{\Delta n}$ is the temperature at a position located at a distance Δn (the reflection is scored). In probability terms, equation (10) states that if a particle is at a boundary having a prescribed heat flux, it is reflected back into the solid by distance Δn and the quantity $(\frac{q}{k})\Delta n + (\frac{S}{2k})\Delta n^2$ is scored.

This type of boundary is known as a reflecting barrier.

Lastly, consider a boundary having a convective boundary condition

$$-K\frac{\partial T}{\partial n} = h(T_a - T_w) \qquad (11)$$

where h is the heat transfer coefficient and T_a is the ambient temperature.

Imploring the Taylor's series again, it turns out that

$$T_w = \frac{1}{1+h\Delta n/k}T_{\Delta n} + \frac{h\Delta n/k}{1+h\Delta n/k} + \frac{S\Delta n^2/2k}{1+h\Delta n/k} \qquad (12)$$

The interpretation is that if a particle is at the boundary, it has the probability $\frac{1}{1+h\Delta n/k}$ of being reflected and a probability $\frac{h\Delta n/k}{1+h\Delta n/k}$ of being absorbed with T tallied. If there is a heat source, a quantity $\frac{S\Delta n^2/2k}{1+h\Delta n/k}$ is tallied in either case. Simply, the convection boundary condition is equivalent to a partially absorbing and partially reflecting barrier.

Procedure and Computational Experiments

The Computer Programs

The computer programs were developed using VAX FORTRAN on a VAX 4.4 system.

Case 1: Steady-state solution for a 2-dimensional arbitrarily shaped body with prescribed temperature at the boundary using Monte Carlo – Fixed random walk method.
Case 2: Steady-state solution for a 2-dimensional arbitrarily shaped body with prescribed temperature using the Monte Carlo- Floating random walk method.

The Random Number Generator

Since the basic procedure of the Monte Carlo method is the manipulation of random numbers, its generation is vital to the success of the investigation.

For this study, an in-built subroutine, RAN on the VAX 4.4 system was employed. There are no restrictions on the seed used, although it should be initialized with different values on separate runs. The seed is updated automatically, and RAN uses the following algorithm to update the seed passes as the parameter.
SEED = 69069 * SEED + (MOD 2 **32)

For the programs; the input parameters are the external boundary nodes and the corresponding temperatures, the co-ordinates of the seed for the random number generator and the values of the thermal conductivity and the heat generation term. The programs then compute the following, namely; the average number of steps per walks, the number of steps in the longest walk, the number of steps in the shortest walk and the temperature at the departure point.
A parametric study was carried out as follows, Ogunjobi (1992):
The seed for the random number was varied arbitrarily and the effect on the output parameters was observed. The seed, the number of walks and the step size were also varied. The results are given in the Tables 1, 2, 3 and 4.

Table 1: Variation of Random Number Generator Seed for a Fixed Random Walk
Step Size = 0.02; Number of Walk = 1000

Departure Point	Seed	Minimum Step	Maximum Step	Average Step	Temperature (^{0}K)
0.28,0.30	45678340	11	554	85.63	507.38
	154	11	407	86.55	506.42
	33333	12	489	84.08	507.15
	9851	14	490	86.90	506.23
	787134	10	406	88.66	506.80
	51	11	467	90.19	506.88

Table 2: Variation of Number of Walks for a Fixed Random Walk
Step Size = 0.02

Departure Point	Seed	Number Walks	Minimum Step	Maximum Step	Average Step	Temperature (^{0}K)
0.28,0.30	51	100	13	268	87.48	506.90
	4447	200	13	330	90.79	508.69
	12345	400	11	365	82.97	507.73
	999	500	11	354	82.92	506.85
	645329	800	9	454	89.88	506.54
	2	1000	12	398	87.92	506.84

Table 3: Variation of Step Size for a Fixed Random Walk
Number of Walk = 1000

Departure Point	Seed	Minimum Step	Maximum Step	Average Step	Step Size	Temp. (^{0}K)
0.24,0.24	75311	6	398	68.67	0.02	511.02
	537	3	104	13.88	0.04	510.21
	124904	2	52	7.87	0.06	509.92
	8576	1	16	3.13	0.08	514.97

Table 4: Variation of Random Number Generator Seed for a Floating Random Walk

Number of Walk = 1000

Departure Point	Seed	Minimum Step	Maximum Step	Average Step	Temperature (^{0}K)
0.28,0.30	45678340	2	28	6.49	506.43
	154	2	31	6.35	506.71
	33333	2	31	6.70	506.62
	9851	2	26	6.35	507.62
	787134	2	29	6.55	506.71
	51	2	31	6.66	507.78

Parameters:

Heat generation rate - 0.0W/m^3

Thermal conductivity - 356.78 W/mK

Comparison with Finite Difference Method

Consider a 2-dimensional problem below, where temperatures at points T_1, T_2, T_3 and T_4 are required.

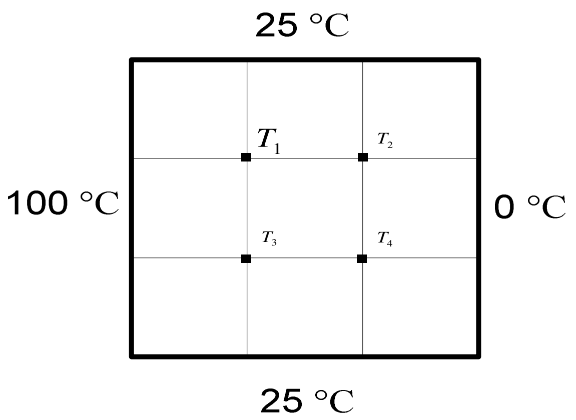

Figure 5: A 2-dimensional heat conduction problem

The solution of the problem in Figure 5 using the Finite Difference with Gauss-Seidel iteration and the Fixed Walk Monte Carlo method are given in table 5.

Table 5

Method	T_1 (ºC)	T_2 (ºC)	T_3 (ºC)	T_4 (ºC)
Finite Difference (with Gauss-Seidel iteration)	50.00	25.00	50.00	25.00
Monte Carlo (Fixed Walk) method	50.58	24.58	50.58	24.68

Results and Discussion

After conducting some computational experiments, it was obtained that the variation of the random number generator seed doesn't affect the departure point temperature within statistical error limit.

It was also observed that as the number of walks is varied, the departure point temperature also changes, it is apparent that the finer the mesh, the more accurate the solution would be, but the longer the computational time to solve the problem. Also, the maximum number of steps, the minimum number of steps and the average number of steps per walk greatly reduced for the floating random walk compared to the fixed random walk for the same solution. The fixed random walk and the floating random walk methods gave comparable results of 507.26^0K and 506.98^0K respectively for temperature at a point. The Monte Carlo method also gave comparable results with the Finite Difference Method (FDM) (using Gauss- Seidel iteration) for a 2-dimensional regular shaped heat conduction problem. A sample of the external boundary nodes and the associated prescribed temperatures is shown in Appendix I.

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Conclusion

The Monte Carlo algorithm requires no discretization of the surface of the problem domain; hence the memory requirements are expected to be lower than of approaches based on spatial discretization, such as the FDM. The Monte Carlo method can deal more comfortably with irregular shaped geometries and boundary conditions.

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APPENDIX I

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$
$ type fas1c.out
          EXTERNAL BOUNDARY NODE
```

S/N	X-COORD		Y-COORD	TEMP/deg
1	0.0400	,	0.2000	500.0200
2	0.0400	,	0.1800	500.0400
3	0.0400	,	0.1600	500.0400
4	0.0600	,	0.1600	490.0300
5	0.0600	,	0.1400	490.0000
6	0.0800	,	0.1400	490.5000
7	0.1000	,	0.1400	490.0000
8	0.1200	,	0.1400	480.0500
9	0.1400	,	0.1400	500.0000
10	0.1600	,	0.1400	512.0000
11	0.1800	,	0.1400	512.0500
12	0.2000	,	0.1400	512.1000
13	0.2200	,	0.1400	513.0000
14	0.2400	,	0.1400	512.3400
15	0.2600	,	0.1400	534.0000
16	0.2600	,	0.1200	532.0000
17	0.2800	,	0.1200	515.8000

18	0.2800	,	0.1000	529.0000
19	0.3000	,	0.1000	532.7000
20	0.3200	,	0.1000	530.0000
21	0.3200	,	0.0800	534.2000
22	0.3400	,	0.0800	538.9000
23	0.3400	,	0.0600	523.4000
24	0.3600	,	0.0600	532.9000
25	0.3600	,	0.0800	523.4000
26	0.3800	,	0.0800	528.9200
27	0.4000	,	0.0800	518.9000
28	0.4000	,	0.1000	518.9000
29	0.4000	,	0.1200	519.9000
30	0.4000	,	0.1400	520.0000
31	0.4000	,	0.1600	513.4800
32	0.4000	,	0.1800	521.0000
33	0.4000	,	0.2000	510.0000
34	0.4200	,	0.2000	509.8800
35	0.4200	,	0.2200	507.6500
36	0.4200	,	0.2400	504.8200
37	0.4200	,	0.2600	490.9900
38	0.4400	,	0.2600	491.6600

39	0.4400	,	0.2800	490.1300
40	0.4600	,	0.2800	498.7700
41	0.4600	,	0.3000	496.0000
42	0.4800	,	0.3000	482.9900
43	0.4800	,	0.3200	481.2300
44	0.4800	,	0.3400	486.5700
45	0.4800	,	0.3600	472.4300
46	0.4600	,	0.3600	476.5300
47	0.4600	,	0.3800	465.8900
48	0.4400	,	0.3800	467.8700
49	0.4400	,	0.4000	478.9000
50	0.4200	,	0.4000	478.9200
51	0.4200	,	0.4200	478.2900
52	0.4000	,	0.4200	489.0000
53	0.4000	,	0.4400	484.5800
54	0.4000	,	0.4600	480.0000
55	0.4000	,	0.4800	480.2300
56	0.4000	,	0.5000	487.0000
57	0.4000	,	0.5200	487.2300
58	0.4000	,	0.5400	479.9800
59	0.3800	,	0.5400	478.3900
60	0.3600	,	0.5400	478.8200

61	0.3600	,	0.5600	489.0000
62	0.3400	,	0.5600	489.7600
63	0.3200	,	0.5600	489.6700
64	0.3000	,	0.5600	490.0700
65	0.2800	,	0.5600	498.7000
66	0.2600	,	0.5600	494.3500
67	0.2400	,	0.5600	498.7600
68	0.2200	,	0.5600	500.1900
69	0.2200	,	0.5400	500.9800
70	0.2000	,	0.5400	500.7500
71	0.1800	,	0.5400	501.3400
72	0.1800	,	0.5200	507.8400
73	0.1600	,	0.5200	504.3600
74	0.1600	,	0.5000	512.9000
75	0.1400	,	0.5000	518.9900
76	0.1400	,	0.4800	524.3300
77	0.1400	,	0.4600	538.9000
78	0.1400	,	0.4400	534.7800
79	0.1400	,	0.4200	530.3900
80	0.1200	,	0.4200	543.2100
81	0.1200	,	0.4000	540.9800

82	0.1200	,	0.3800	534.7800
83	0.1200	,	0.3600	523.4800
84	0.1200	,	0.3400	523.4500
85	0.1200	,	0.3200	512.3400
86	0.1200	,	0.3000	523.1400
87	0.1000	,	0.3000	512.3400
88	0.1000	,	0.2800	512.3400
89	0.1000	,	0.2600	512.3400
90	0.0800	,	0.2600	503.2000
91	0.0800	,	0.2400	490.0000
92	0.0600	,	0.2400	501.9900
93	0.0600	,	0.2200	509.7300
94	0.0400	,	0.2200	500.0000

HEAT GENERATION RATE = 0.00000 W/m**3

THERMAL CONDUCTIVITY = 356.78998 W/m*k

DEPARTURE POINT = 0.28000 , 0.30000

NO OF WALKS PERMITTED = 1000

COMPUTATIONAL RESULTS :

SEED FOR RANDOM NO GENERATOR = 154

AVERAGE NO STEPS PER WALK = 6.349

NO OF STEPS IN THE LONGEST WALK = 31

NO OF STEPS IN THE SHORTEST WALK = 2

TEMPERATURE AT THE DEPARTURE POINT = 506.708 K

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